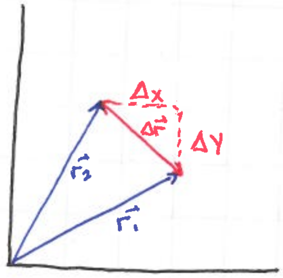
Ch.4 + RECAP

February 21, 2019

|| Chapter 4: Kinematics in Two Dimensions ||

	One Dimension ($\vec{x}$)	Two-Dimensions
Position	x_0 or x	$\vec{r} = x\hat{i} - y\hat{j}$
displacement	$\Delta x = x - x_0$	$\Delta r = \Delta x\hat{i} + \Delta y\hat{j}$
ave. velocity	$V_{avg} = \frac{\Delta x}{\Delta t}$	$V_{avg.} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$
Instantaneous Velocity	$V_x = dx/dt$ $= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta y}$	$V = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = V_x\hat{i} + V_y\hat{j}$ $V_x = \frac{dx}{dt}$; $V_y = \frac{dy}{dt}$
avg. acceleration	$a_{avg,x} = \frac{\Delta V_x}{\Delta t}$	$a_{avg.} = \frac{\Delta V_x}{\Delta t}\hat{i} + \frac{\Delta V_y}{\Delta t}\hat{j}$
Inst. acceleration	$a_x = \frac{dv_x}{dt} = \lim_{\Delta t \to 0} \frac{\Delta V_x}{\Delta t}$	

RECAP



$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$$
$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$$
$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$$

$$\vec{V} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = V_x\hat{i} + V_y\hat{j}$$

$$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

* In two-dimensions, the x- and y- components are independent & can be treated separately.

→ TRUE of any vectors

x-component	y-component
$V_x = V_{0x} + a_x t$	$V_y = V_{0y} + a_y t$
$x - x_0 = V_{0x} t + \frac{1}{2} a_x t^2$	$y = y_0 + V_{0y} t + \frac{1}{2} a_y t^2$
$V_x^2 = V_{0x}^2 + 2a_x(x - x_0)$	$V_y^2 = V_{0y}^2 + 2a_y(y - y_0)$

* above equations apply only if $\vec{a}$ is constant (1-D)

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Projectile Motion

↳ Two-Dimensional motion in which gravity is the only force acting

$\boxed{a_x = 0\ m/s^2}$ $\boxed{a_y = -g = -9.80\ m/s^2}$

Equations for Projectile Motion

Vertical motion $\boxed{a_y = -g = -9.80\ m/s^2}$ Horizontal motion $\boxed{a_x = 0\ m/s^2}$

$V_y = V_{0y} + a_x t$

$Y = Y_0 + V_{0y}t + \frac{1}{2}a_y t^2$

$V_y^2 = V_{0y}^2 + 2a_y(y - y_0)$

$V_x = V_{0x}$

$x = x_0 + V_{0x}t$

$V_x^2 = V_{0x}^2$

} only two equations in x-direction for projectile motion

Vertical Motion

Y

Y_0

V_{0y}

V_y

$a_y = -9.80\ m/s^2$

t

Horizontal motion

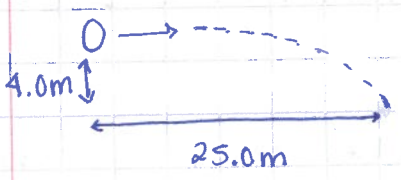
x_0

x

V_{0x}

t

Problem 4.17



$x_0 = 0m$

$x = 25m$

$V_{0x} =$

$t =$

$Y = 4.0m$

$Y = 0m$

$V_{0y} = 0\ m/s$

$V_y =$

$a_y = -9.80\ m/s$

$t =$

$Y = \overset{=0}{Y_0} + \overset{=0}{V_{0y}t} + \frac{1}{2}a_y t^2$

$0 = Y_0 + \frac{1}{2}a_y t^2$

$t = \sqrt{\frac{-2Y_0}{a_y}} = \sqrt{\frac{-2(4.0\ m)}{-9.80\ m/s^2}} \Rightarrow t = 0.904\ s$

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Problem 4.17 continued

$x = \overset{=0}{x_0} + V_{0x} t$

$x = V_{0x} t$

$V_{0x} = \frac{x}{t} = \frac{25m}{0.904s}$

$V_{0x} = 27.7 m/s \Rightarrow \boxed{V_{0x} = 28 m/s}$

Projectile Motion

$V_y = 0 m/s$ at the highest point

$\boxed{\text{Speed } V = \sqrt{V_x^2 + V_y^2}}$

$\boxed{\begin{gathered}\text{Direction of travel} \\ \theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)\end{gathered}}$

$\boxed{\begin{aligned} V_{0x} &= V_0 \cos\theta \\ V_{0y} &= V_0 \sin\theta \end{aligned}}$

*Motion is symmetric

→ Speed at a given height is the same

→ if $y = y_0$, then time up = time down

Common Q's

1 – How fast does it travel Horizontally?

2 – How high does it rise?

3 – What is the min speed while it is in the air?

4 – What is it's speed & direction of travel at t = 2.0s?

How to approach Q's

1- get time in air (if $y = y_0$, get time to highest point & double)
→ then use $x = x_0 + V_{0x} t$

2- what is y when $v_y = 0 m/s$?

3- it occurs at the highest point when $v_y = 0 m/s$
$\boxed{V_{min} = V_{0x} = V_0 \cos\theta}$

4 - You need v_y and v_x at t = 2.0s $\boxed{V = \sqrt{V_x^2 + V_y^2}}$ $\boxed{\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)}$

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1Q:

A leopard springs upward at a 55° angle and then falls back to the ground. Does the leopard, at any point on its trajectory, ever have a speed that is one-half of its initial Value?

NO

$V_0 = 10.0$ m/s

$\theta = 55°$

$V_{0x} = V_{0x} \cos 55° = 5.7$ m/s

Q: What is height of projectile when it has traveled $x = 15.0$ m?

↳ How long does it take to travel $\Delta x = 15.0$m Horizontally. What is height at that time?

	CONSTANT	ZERO
x		
Y		
r		
Vx	✓	
Vy		
ax	✓	✓
ay	✓	

Q: at what time is the speed = 30.0 m/s?

$V = \sqrt{V_x^2 + V_y^2}$

- figure out what V_y will give you that Speed and solve for when it has that speed

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	CONSTANT	ZERO
x		
y		
r		
V_x	✓	
V_y		
a_x	✓	✓
a_y	✓	

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$V = \sqrt{V_x^2 + V_y^2}$

- figure out what V_y will give you that speed and solve for when it has that speed

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- constant acceleration in two dimensions:

$$\boxed{\begin{aligned} V_x &= V_{0x} + a_x t \\ x &= x_0 + V_{0x} t + \tfrac{1}{2} a_x t^2 \\ V_x^2 &+ V_{0x}^2 + 2a_x (x - x_0) \end{aligned}}$$

$$V_y = V_{0y} + a_y t$$

$$y = y_0 + V_{0y} t + \tfrac{1}{2} a_y t^2$$

$$V_y^2 = V_{0y}^2 + 2a_y (y - y_0)$$

Recap + ch 4

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Projectile motion $\rightarrow a_x = 0 m/s^2$
$\rightarrow a_y = -g = -9.80 m/s^2$

Vertical Motion:

$y_0 =$

$y =$

$V_{0y}:$

$V_y:$

$a_y = -9.80 m/s^2$

$t =$

$$\boxed{\begin{aligned} V_y &= V_{0y} + a_y t \\ y &= y_0 + V_{0y} t + 1/2\, a_y t^2 \\ V_y{}^2 &= V_{0y}{}^2 + 2a_y(y - y_0) \end{aligned}}$$

Horizontal motion

$x_0 =$

$x =$

$V_{0x} =$

$t =$

$V_x = V_{0x}$

$$\boxed{x = x_0 + V_{0x} t}$$

$$\boxed{\begin{aligned} V_{0x} &= V_0 \cos\theta \\ V_{0y} &= V_0 \sin\theta \end{aligned}}$$

Ex

A projectile is launched at 35.0 m/s at a 50.0° angle. & hits the ground at the same height.

- a) How far does the projectile travel? b) How high does it rise?
- c) what is the minimum speed while in the air? 22.5 m/s
- d) when is the speed = 28.0 m/s ?
- e) what angle does it hit the ground at

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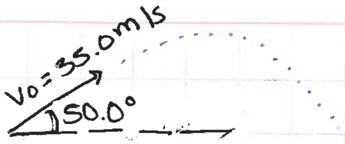

$V_{ox} = (35.0 m/s) \cos(50.0°) = 22.5 m/s$

$V_{oy} = (35.0 m/s \sin(50.0°) = 26.8 m/s$

c) $22.5 m/s = V_{min} = V_{ox}$

a) To get time in air:

1) Get time to highest point ($v_y = 0 m/s$) & double

2) get time $y = y_0$

3) Get time for $v_y = -26.8 m/s$

$v = \sqrt{V_x^2 + V_y^2}$

$x = 0 m$

$x = ?$

$V_{ox} = 22.5 m/s$

$t = ?$

$y_0 =$

$y =$

$V_{oy} = 26.8 m/s$

$V_y = 0 m/s$

$a_y = -9.80 m/s^2$

$t =$

a) $\overset{=0}{V_y} = V_{oy} + a_y t$

$0 = V_{oy} + a_y t$

$t = \frac{-V_{oy}}{a_y} = \frac{-26.8 m/s}{-9.80 m/s} \longrightarrow t = 2.74 s$ to highest point

Total time in air $= 2(2.74 s) = \underline{5.48 s}$

5.48s

$x = \overset{=0}{x_0} + V_{ox} t$

$x = V_{ox} t$

$x = (22.5 m/s)(5.48 s)$

$x = 123 m$

a) 123m (How far does the projectile travel?)

b) $Y_0 = 0 m$

$Y = ?$

$V_{oy} = 26.8 m/s$

$V_y = 0 m/s$ at highest point

$a_y = -9.80 m/s^2$

$t = 2.74 s$ to highest point

$y = \overset{=0}{y_0} + V_{oy} t + \frac{1}{2} a_y t^2$

$y = V_{oy} t + \frac{1}{2} a_y t^2$

$y = (26.8 m/s)(2.74 s) + \frac{1}{2}(-9.80 m/s^2)(2.74 s)^2$

$y = 36.6 m$

b) 36.6m (How high does it rise?)

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$V_y^2 = V_{oy}^2 + 2a_y(y - y_o)$

d) Figure out what Vy will give you $v = 28.0\,m/s$.
Then find t to reach v_y.

$V^2 = V_x^2 + V_y^2 \rightarrow V_y^2 = V^2 - V_x^2 \rightarrow V_y = \pm\sqrt{V^2 - V_x^2}$

$V_y = \pm\sqrt{V^2 - V_x^2}$

$V_y = \pm\sqrt{(28.0\,m/s)^2 - (22.5\,m/s)^2}$

$V_y = \pm 16.7\,m/s$

$V_y = V_{oy} + a_y t \longrightarrow t = \dfrac{V_y - V_{oy}}{a_y}$

$t = \dfrac{\pm 16.7\,m/s - 26.8\,m/s}{-9.80\,m/s^2}$

$t = 4.44s$ and $t = 1.03s$

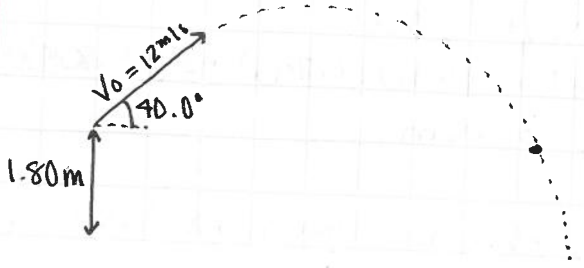
d) $t = 4.44s$ and $t = 1.03s$ (when is the speed = 28.0m/s?)

e) what angle does it hit the ground at?

$\theta = \tan^{-1}\left(\dfrac{V_y}{V_x}\right)$

$\theta = -50°$

Problem 4.15



$Y_o = 1.80\,m$

$Y = 0\,m$

$V_{oy} = (12.0\,m/s)\sin 40.0 = 7.71\,m/s$

$V_y = ?$

$a_y = -9.80\,m/s^2$

$t = ?$

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$\overset{=0}{Y} = y_0 + V_{oy} t + 1/2 \, a_y t^2$

$\frac{1}{2} a_y t^2 + V_{oy} t + y_0 = 0 \qquad At^2 + Bt + C = 0$

- $A = \frac{1}{2} a_y = -4.90 \, m/s^2$
- $B = v_{oy} = 7.71 \, m/s$
- $C = y_0 = 1.80 \, m$

$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \qquad \underline{t = 1.78s} \;\; \& \;\; t < 0s$

$X_0 = 0m$

$x = ?$

$V_{ox} = (12.0 \, m/s) \cos 40.0° = 9.19 \, m/s$

$t = 1.78s$

$x = \overset{=0}{x_0} + V_{ox} t$

$x = V_{ox} t$

$x = (9.19 m/s)(1.78s)$

$\boxed{x = 16.4m}$

Problem 4-D

$V_{ox} = 19 m/s$

$t = 5.0s$

$x = \overset{=0}{x_0} + V_{ox} t$

$= (19 m/s)(5.0s)$

$\underline{X = 95m}$

$V_{oy} = ?$

$V_y = 0 m/s$ to highest point

$a_y = -9.80 \, m/s^2$

$t = 2.5s$ to highest point

$\overset{=0}{V_y} = V_{oy} + a_y t$

$V_{oy} = -a_y t$

$= -(-9.80 \, m/s^2)(2.50s)$

$= 24.5 m/s$

$Y = \overset{=0}{y_0} + V_{oy} t + 1/2 \, a_y t^2$

$y = (24.5 m/s)(2.5s) + \swarrow$

$1/2 \, (-9.80 m/s^2)(2.5s)^2$

$\boxed{y = 31m}$

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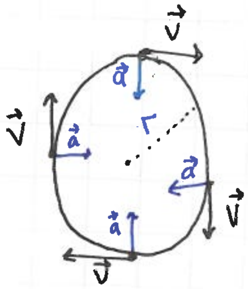
Centripetal Acceleration

Uniform circular motion → object moving in a circular path of radius r at a constant speed v

↓ object is accelerating because the direction is changing



Period T ⟹ time for one complete revolution

Given T and r, what is speed v? $v = \frac{2\pi r}{T}$

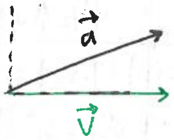
Given without derivation:

- Centripetal acceleration: ↳ acceleration due to a change in direction

$$\boxed{a = \frac{v^2}{r}}$$

v → speed in m/s
r → radius in m

* direction is always towards the center of the circular path.



what is the object doing?

Speeding up + turning upwards

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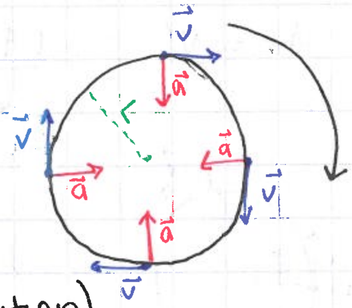
tangential acceleration a_t → same or opposite directions
as $\vec{v}$ ↓ speed up ↘ slows down

Radial acceleration a_r → ⊥ to the $\vec{v}$
(centripetal acceleration) ↳ causes a change in direction

RECAP

⟹ Projectile motion examples
⟹ Uniform circular motion

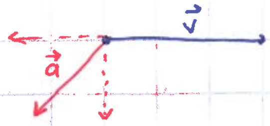
$$\boxed{V = \frac{2\pi r}{T}}$$

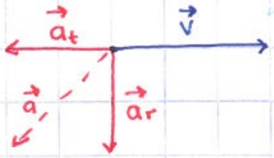


Centripetal acceleration (Radial acceleration)

$$\boxed{a_r = \frac{V^2}{r}}$$ → direction is always towards the center of the circular path.
↳ magnitude

* $\vec{a}_r$ is the acceleration because of a change in direction
($\vec{a}_r$ is ⊥ to $\vec{v}$)
* $\vec{a}_t$ is the acceleration because of a change in speed
(a_t is // to $\vec{v}$)





⇒ Speed is decreasing because $\vec{a}_t$ is in opposite direction as $\vec{v}$

⇒ Direction is changing, object is turning downwards.

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PROBLEM 4.E

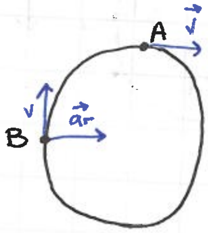
$r = 6.37 \times 10^6 m$

$T = 86,400s$

$$v = \frac{2\pi r}{T} = \frac{2\pi(6.37\times10^6 m)}{86,400s} \rightarrow \boxed{v = 463 m/s}$$

$$a_r = \frac{v^2}{r} = \frac{(463 m/s)^2}{6.37\times10^6 m} \longrightarrow \boxed{a_r = 3.4\times10^{-2} m/s^2}$$

↓ Too small to sense



$v = 6 m/s$

$r = 3m$

$$a_r = \frac{(6 m/s)^2}{3m}$$

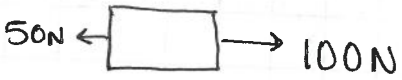
$$= 12 m/s^2$$

||Chapter 5: Force & Motion||

Ch. 2 & 4 → looked at motion w/o studying what caused the motion

* Object accelerates because of a net force ($\Sigma\vec{F}$ or $\vec{F}_{net}$)

↓ Vector sum of all forces



Net force of 50N to the right

↳ The object is accelerating to the right